Superallowed Fermi Beta Decay and Coulomb Mixing in Nuclei

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Abstract. Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay provides a direct measure of the weak vector coupling constant, $G_{\rm V}$. We survey current world data on the nine accurately determined transitions of this type, which range from the decay of ^{10}C to that of ^{54}Co, and demonstrate that the results confirm conservation of the weak vector current (CVC) but differ at the 98% confidence level from the unitarity condition for the Cabibbo-Kobayashi-Maskawa (CKM) matrix. We examine the reliability of the small calculated corrections that have been applied to the data, and conclude that there are no evident defects although the Coulomb correction, δ_C, depends sensitively on nuclear structure and thus needs to be constrained independently. The potential importance of a result in disagreement with unitarity, clearly indicates the need for further work to confirm or deny the discrepancy. We examine the options and recommend priorities for new experiments and improved calculations. Some of the required experiments depend upon the availability of intense radioactive beams. Others are possible with existing facilities.

INTRODUCTION

In probing the properties of the weak interaction, superallowed $0^+ \rightarrow 0^+$ nuclear beta decay has become a singularly valuable tool, principally because of its relative independence from the effects of nuclear structure, which are notoriously difficult to account for with high accuracy. Since the measured $0^+ \rightarrow 0^+$ transitions are between $T = 1$ analog states, nuclear-structure effects only enter at the level of the differences between the parent and daughter wave functions. These differences, which are caused by Coulomb and charge-dependent nuclear forces, turn out to be very small, and introduce a correction of order 1% when the experimental ft-values are used to extract a value for the effective weak vector coupling constant, $G'_{\rm V}$. Even a conservative estimate of the uncertainties in this correction indicate that structure-dependent uncertainties should not afflict the experimental determination of $G'_{\rm V}$ above the level of approximately $\pm 0.1\%$. As a result, considerable effort has gone into making ft-value measurements that achieve this level of experimental precision or better.

In this paper, we begin by summarizing the current status of world data on superallowed Fermi beta decays and demonstrating the extent to which these data test the Conserved Vector Current (CVC) hypothesis and the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In fact, we show that the success of the measurements has been such that experiment has outstripped theory. The theoretical uncertainties in calculated corrections – including the nuclear-structure-dependent charge-corrections – are now the limiting factor when superallowed beta decay is used to test the validity of the Standard Model. On the one hand, this can be viewed as a serious limitation to the continued usefulness of such measurements in future or, at least, as an indication that any improvements in their precision will have to be used to probe charge-dependence in nuclear structure rather than the fundamental properties of weak interactions. On the other hand, it can also be taken as a challenge to nuclear-structure theorists and experimenters alike to establish the effects of charge-dependence by independent means and thus to improve our ability overall to calculate them precisely.

Following our survey of world data, we outline the current approach to calculating charge-dependent effects, describe measurements that have been used to test these calculations independently, and consider future

prospects for improved results. In particular, we shall consider the prospects for enlarging the sample of well-measured $0^+, T = 1$ superallowed emitters, and examine what needs to be done before such measurements can usefully contribute to our fundamental understanding of the weak interaction.

CURRENT STATUS OF WORLD DATA

Superallowed Fermi $0^+ \rightarrow 0^+$ nuclear beta decays [1,2] provide both the best test of the CVC hypothesis in weak interactions and, together with the muon lifetime, the most accurate value for the up-down quark-mixing matrix element of the CKM matrix, V_{ud}. Because the axial current cannot contribute in lowest order to transitions between spin-0 states, the experimental ft-value is related directly to the vector coupling constant. Specifically, for an isospin-1 multiplet,

$$ft(1+\delta_R) = \frac{K}{{G'_{\mathrm{V}}}^2 \langle M_{\mathrm{V}} \rangle^2}, \tag{1}$$

with

$$\begin{aligned} G'_{\mathrm{V}} &= G_{\mathrm{V}}(1+\Delta_{\mathrm{R}}^{\mathrm{V}})^{1/2}, \\ \langle M_{\mathrm{V}} \rangle^2 &= 2(1-\delta_C), \\ K/(\hbar c)^6 &= 2\pi^3 \hbar \ln 2/(m_e c^2)^5 = (8120.271 \pm 0.012) \times 10^{-10} GeV^{-4} s, \end{aligned} \tag{2}$$

where f is the statistical rate function, t is the partial half-life for the transition, $\langle M_{\mathrm{V}} \rangle$ is the Fermi matrix element and G_{V} is the primitive vector coupling constant. The physical constants used to evaluate K were taken from the most recent Particle Data Group publication [3]. These equations also include three calculated correction terms – all of order 1%. We write δ_R as the nucleus-dependent part of the radiative correction, $\Delta_{\mathrm{R}}^{\mathrm{V}}$ as the nucleus-independent part of the radiative correction, and δ_C as the isospin symmetry-breaking correction. A general description of these three correction terms and the methods used in their calculation has appeared elsewhere [1,2,4]. In the present context, it is sufficient to note that nuclear structure plays a small role in the determination of δ_R, but it is predominant for that of δ_C.

Equations (1) and (2) can now be combined into a form that is convenient for the analysis of experimental results:

$$\mathcal{F}t \equiv ft(1+\delta_R)(1-\delta_C) = \frac{K}{2G_{\mathrm{V}}^2(1+\Delta_{\mathrm{R}}^{\mathrm{V}})}. \tag{3}$$

Here we have defined $\mathcal{F}t$ as the "corrected" ft-value. From this equation, it is evident that the $\mathcal{F}t$-values obtained from $0^+ \rightarrow 0^+$ transitions in different nuclei can constitute a stringent test of CVC, which requires them all to be equal.

To date, superallowed $0^+ \rightarrow 0^+$ transitions have been measured to $\pm 0.1\%$ precision or better in the decays of nine nuclei ranging from ^{10}C to ^{54}Co. World data on Q-values, lifetimes and branching ratios – the results of over 100 independent measurements – were thoroughly surveyed [1] in 1989 and then updated several times since, most recently for the WEIN98 conference [4]. The resulting weighted averages are given in the first three columns of Table 1. Using the calculated electron-capture probabilities [1] given in the next column, we obtain the "uncorrected" ft-values listed in column 5 with partial half-lives determined from the formula $t = t_{1/2}(1 + P_{EC})/R$.

To convert these results for ft into $\mathcal{F}t$-values, we apply the δ_C and δ_R corrections given in columns 6 and 7. We describe the δ_C calculations and their dependence on nuclear structure in a later section of this paper; the δ_C values used in Table 1 were taken from the last column of Table 3. The δ_R values come from our previous recent analyses [2,4] and arise from a variety of primary sources [1,5–8]. It is important to appreciate that the values of δ_C and δ_R result from more than one independent calculation. In the case of δ_R, the calculations are in complete accord with one another; for δ_C, we have used an average of two independent calculations with assigned uncertainties that reflect the (small) scatter between them. Thus, in a real sense, both experimentally and theoretically, the $\mathcal{F}t$-values given in Table 1 and plotted in Fig. 1 represent the totality of current world knowledge. The uncertainties reflect the experimental uncertainties and an estimate of the *relative* theoretical uncertainties in δ_C. There is no statistically significant evidence of inconsistencies in the data ($\chi^2/\nu = 1.1$), thus verifying the expectation of CVC at the level of 3×10^{-4}, the fractional uncertainty quoted on the average $\mathcal{F}t$-value.

TABLE 1. Experimental results (Q_{EC}, $t_{1/2}$ and branching ratio, R), electron-capture probabilities (P_{EC}) and calculated corrections (δ_R and δ_C) for $0^+ \to 0^+$ transitions.

	Q_{EC} (keV)	$t_{1/2}$ (ms)	R (%)	P_{EC} (%)	ft (s)	δ_C (%)	δ_R (%)	$\mathcal{F}t$ (s)
^{10}C	1907.77(9)	19290(12)	1.4645(19)	0.296	3038.7(45)	0.16(3)	1.30(4)	3072.9(48)
^{14}O	2830.51(22)	70603(18)	99.336(10)	0.087	3038.1(18)	0.22(3)	1.26(5)	3069.7(26)
^{26m}Al	4232.42(35)	6344.9(19)	$\geq$ 99.97	0.083	3035.8(17)	0.31(3)	1.45(2)	3070.0(21)
^{34}Cl	5491.71(22)	1525.76(88)	$\geq$ 99.988	0.078	3048.4(19)	0.61(3)	1.33(3)	3070.1(24)
^{38m}K	6044.34(12)	923.95(64)	$\geq$ 99.998	0.082	3049.5(21)	0.62(3)	1.33(4)	3071.1(27)
^{42}Sc	6425.58(28)	680.72(26)	99.9941(14)	0.095	3045.1(14)	0.41(3)	1.47(5)	3077.3(23)
^{46}V	7050.63(69)	422.51(11)	99.9848(13)	0.096	3044.6(18)	0.41(3)	1.40(6)	3074.4(27)
^{50}Mn	7632.39(28)	283.25(14)	99.942(3)	0.100	3043.7(16)	0.41(3)	1.40(7)	3073.8(27)
^{54}Co	8242.56(28)	193.270(63)	99.9955(6)	0.104	3045.8(11)	0.52(3)	1.40(7)	3072.2(27)
						Average, $\overline{\mathcal{F}t}$		3072.3(9)
						χ^2/ν		1.10

In using this average $\mathcal{F}t$-value to determine V_{ud} and test CKM unitarity, we must account for additional uncertainty: *viz*

$$\overline{\mathcal{F}t} = 3072.3 \pm 0.9 \pm 1.1, \tag{4}$$

where the first error is the statistical error of the fit (as illustrated in Fig. 1), and the second is an error related to the systematic difference between the two calculations of δ_C by Towner, Hardy and Harvey [9,10] and by Ormand and Brown [11] that we have combined in reaching this result. (For a more complete discussion of how we treat these theoretical uncertainties, see reference [1].) We now add the two errors linearly to obtain the value we use in subsequent analysis:

$$\overline{\mathcal{F}t} = 3072.3 \pm 2.0. \tag{5}$$

The value of V_{ud} is obtained by relating the vector constant, $G_{\rm V}$, determined from this $\overline{\mathcal{F}t}$ value, to the weak coupling constant from muon decay, $G_{\rm F}/(\hbar c)^3 = (1.16639 \pm 0.00001) \times 10^{-5}$ GeV^{-2}, according to:

$$V_{ud}^2 = \frac{K}{2G_{\rm F}^2(1+\Delta_{\rm R}^{\rm V})\overline{\mathcal{F}t}}. \tag{6}$$

With the nucleus-independent radiative correction adopted from Sirlin [12], $\Delta_{\rm R}^{\rm V} = (2.40 \pm 0.08)\%$, we obtain the result

$$\mid V_{ud} \mid = 0.9740 \pm 0.0005, \tag{7}$$

We are now in a position to test the unitarity of the CKM matrix by evaluating the sum of squares of the elements in its first row. With the value just obtained for V_{ud}, combined with the values of V_{us} and V_{ub} quoted by the Particle Data Group [3], the unitarity sum becomes

$$\mid V_{ud} \mid^2 + \mid V_{us} \mid^2 + \mid V_{ub} \mid^2 = 0.9968 \pm 0.0014, \tag{8}$$

which differs from unity at the 98% confidence level.

IS NON-UNITARITY REAL?

The result in equation (8) is a very provocative one. If it is taken at face value, it indicates the need for some extension to the electroweak Standard Model, possibly indicating the presence of right-hand currents or of a

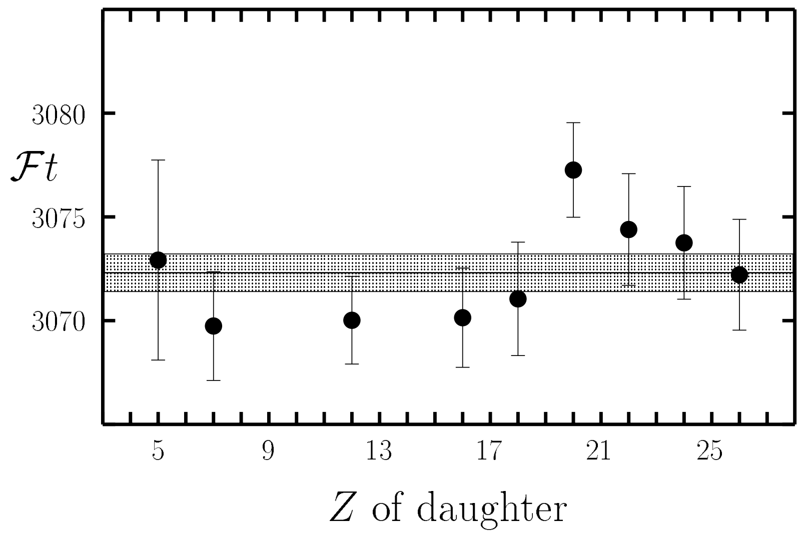

FIGURE 1. $\mathcal{F}t$-values for the nine precision data, and the best least-squares one-parameter fit.

scalar interaction [4]. This would have profound implications. However, the result could have a more trivial explanation. It could instead reflect some undiagnosed inadequacy in the calculated radiative or Coulomb corrections used to evaluate V_{ud} – or possibly a comparable inadequacy in the evaluation of V_{us}. What can be stated with some certainty is that the experimental results for the nine nuclei listed in Table 1 cannot be at fault. Not only do they originate from a large number of independent measurements, but also the error bar associated with $|V_{ud}|$ is *not* predominantly experimental in origin. In fact, if experiment were the sole contributor, the uncertainty would be only ± 0.0001. The largest contributions to the $|V_{ud}|$ error bar come from $\Delta_{\rm R}^{\rm V}$ (± 0.0004) and δ_C (± 0.0003).

Thus, if we are to determine whether the minimal Standard Model has failed, we must eliminate all possible "trivial" explanations for the apparent non-unitarity. To do so, nuclear physicists focus on the reliability of the calculated corrections in V_{ud}. (Others are re-evaluating V_{us} – see reference [4].) But, if there is a fault in the corrections, what size effect are we seeking? To restore unitarity, the calculated radiative corrections (δ_R or $\Delta_{\rm R}^{\rm V}$) for all nine superallowed transitions would all have to be shifted downwards by 0.3%, or the calculated Coulomb corrections, δ_C, all shifted upwards by 0.3%, or some combination of the two. Such changes would constitute a substantial fraction of the total values of these small quantities. We have recently re-examined [4] the calculation of the various correction terms to see whether such large changes are plausible. Our conclusion is largely negative, based on arguments that are now briefly presented.

The radiative correction has been conveniently divided into terms that are nucleus-dependent, δ_R, and terms that are not, $\Delta_{\rm R}^{\rm V}$. These are written

$$\begin{aligned} \delta_R &= \frac{\alpha}{2\pi}\left[\overline{g}(E_m) + \delta_2 + \delta_3 + 2C_{NS}\right] \\ \Delta_{\rm R}^{\rm V} &= \frac{\alpha}{2\pi}\left[4\ln(m_{\rm Z}/m_p) + \ln(m_p/m_{\rm A}) + 2C_{\rm Born}\right] + \cdots , \end{aligned} \tag{9}$$

where the ellipses represent further small terms of order 0.1%. In these equations, E_m is the maximum electron energy in beta decay, $m_{\rm Z}$ the Z-boson mass, $m_{\rm A}$ the a_1-meson mass, and δ_2 and δ_3 the order $Z\alpha^2$ and $Z^2\alpha^3$ contributions. The electron-energy dependent function, $g(E_e, E_m)$, was derived by Sirlin [5]; it is here averaged over the electron spectrum to give $\overline{g}(E_m)$.

Typical values are

$$\delta_R \simeq 0.95 + 0.43 + 0.05 + (\alpha/\pi)C_{NS}\%, \tag{10}$$

where $(\alpha/\pi)C_{NS}$ is of order -0.3% for $T_z = -1$ beta emitters, ^{10}C and ^{14}O, and of order five times smaller for

the $T_z = 0$ emitters, ranging from -0.09% to $+0.03\%$. Thus for $T_z = 0$ emitters $\delta_R \simeq 1.4\%$. If the failure to obtain unitarity in the CKM matrix with V_{ud} from nuclear beta decay is due to the value of δ_R, then δ_R must be reduced to 1.1%. This is not likely. The leading term, 0.95%, involves standard QED and is well verified. The order-$Z\alpha^2$ term, 0.43%, while less secure has been calculated twice [6,7] independently, with results in accord.

For the nucleus-independent term

$$\Delta_{\rm R}^{\rm V} = 2.12 - 0.03 + 0.20 + 0.1\% \quad \simeq \quad 2.4\%, \tag{11}$$

of which the first term, the leading logarithm, is unambiguous. Again, to achieve unitarity of the CKM matrix, $\Delta_{\rm R}^{\rm V}$ would have to be reduced to 2.1%, *i.e.* all terms other than the leading logarithm summing to zero. This also seems unlikely.

Because the leading terms in the radiative corrections are so well founded, attention has focused more on possible weaknesses in the Coulomb correction. Although smaller than the radiative correction, the Coulomb correction is clearly sensitive to nuclear-structure issues. It comes about because Coulomb and charge-dependent nuclear forces destroy isospin symmetry between the initial and final states in superallowed beta-decay. The consequences are twofold: there are different degrees of configuration mixing in the two states, and, because their binding energies are not identical, their radial wave functions differ. Thus, we accommodate both effects by writing $\delta_C = \delta_{C1} + \delta_{C2}$.

There have been several independent calculations of δ_C. The first followed methods developed by Towner, Hardy and Harvey [9] with refinements presented in more recent publications [10,15]. They use shell-model calculations to determine δ_{C1}, and full-parentage expansions in terms of Woods-Saxon radial wave functions to obtain δ_{C2}. A second calculation, by Ormand and Brown [11], also employed the shell model to obtain δ_{C1} but derived δ_{C2} from a self-consistent Hartree-Fock calculation. The results of these two calculations agree remarkably well with one another, the Towner-Hardy-Harvey values being systematically only 0.07% higher than Ormand-Brown ones. It is an average of these two sets of values that we have used for δ_C in our analysis as given in Table 1.

Two more recent calculations provide a valuable check that these δ_C values are not suffering from severe systematic effects. Sagawa, van Giai and Suzuki [13] have added RPA correlations to a Hartree-Fock calculation that incorporates charge-symmetry and charge-independence breaking forces in the mean-field potential to take account of isospin impurity in the core; the correlations, in essence, introduce a coupling to the isovector monopole giant resonance. The calculation is not constrained, however, to reproduce known separation energies as were the two calculations already described. Finally, a large-basis shell-model calculation has been mounted for the $A = 10$ case by Navrátil, Barrett and Ormand [14]. Both of these two new works have produced values of δ_C very similar to, but actually *smaller* than those used in our analysis, *i.e.* worsening rather than helping the unitarity problem.

The typical value of δ_C is of order 0.4%. If the unitarity problem is to be solved by improvements in δ_C, then δ_C has to be raised to around 0.7%. There is no evidence whatsoever for such a shift from recent works. Even so, considerable effort has already gone into making independent experimental checks on the accuracy of the δ_C calculations. These are described in the next section.

TESTS OF THE COULOMB CORRECTION

As shown in equation (2) the Coulomb correction, δ_C, modifies the square of the nuclear matrix element: $|M_{\rm V}|^2 \to |M_{\rm V}|^2(1-\delta_C)$. Here M_V is the Fermi matrix element, the expectation value of the isospin ladder operator, which for isospin $T = 1$ states has the value $M_{\rm V} = \sqrt{2}$. The value of δ_C clearly depends on the detailed structure of the nuclei involved. In the preceding section we have written δ_C as the sum of two components, δ_{C1} and δ_{C2}, the first reflecting the *difference* between configuration mixing in the initial and final states, while the second reflects the differences in their radial wave functions. Our calculations for δ_{C2} have been documented in a paper by Towner, Hardy and Harvey [9] and so will not be repeated here. It is sufficient to stress that constraints are placed on the δ_{C2}-calculations by insisting that the asymptotic forms of the proton and neutron radial functions match the known separation energies. Our calculations for δ_{C1} are documented here.

It is instructive to consider a simplified two-state mixing case as it will illustrate the issues involved. As a specific example, take the case of ^{42}Sc decaying to ^{42}Ca in a superallowed Fermi transition. In the calculation we admit two 0^+ states, the ground state and one excited state. One of these states might be a two-particle

state, $|2p\rangle$, relative to a closed ^{40}Ca core, the other might be a four-particle two-hole state, $|4p\text{-}2h\rangle$. The strong interaction heavily mixes the two states, so the ground state, ψ_0, and excited state, ψ_1, will have wave functions

$$\begin{aligned}\psi_0 &= A|2p\rangle + B|4p\text{-}2h\rangle \\ \psi_1 &= B|2p\rangle - A|4p\text{-}2h\rangle,\end{aligned} \tag{12}$$

where the mixing amplitudes A and B will depend on the details of the strong interaction. The strong interaction, however, is isospin invariant so the same wave functions describe the states in ^{42}Sc and the isospin mirror states in ^{42}Ca. Thus the Fermi matrix element between ground states is

$$\begin{aligned}\langle\psi_0|\tau_+|\psi_0\rangle &= A^2\langle 2p|\tau_+|2p\rangle + B^2\langle 4p\text{-}2h|\tau_+|4p\text{-}2h\rangle \\ &= \sqrt{2}(A^2+B^2) \\ &= \sqrt{2},\end{aligned} \tag{13}$$

while the Fermi matrix element between the ^{42}Sc ground state and the ^{42}Ca excited state is

$$\begin{aligned}\langle\psi_1|\tau_+|\psi_0\rangle &= AB\langle 2p|\tau_+|2p\rangle - AB\langle 4p\text{-}2h|\tau_+|4p\text{-}2h\rangle \\ &= \sqrt{2}(AB-AB) \\ &= 0.\end{aligned} \tag{14}$$

This latter result is just a reminder that the operator for superallowed Fermi decay, being the isospin ladder operator, only connects with states of the same isospin multiplet, *i.e.* analogue states.

Now, suppose we add to the Hamiltonian Coulomb and other charge-dependent forces. These terms, no doubt, will be much weaker than the strong-interaction forces, but their impact is to modify slightly the wave functions ψ_0 and ψ_1 and by differing amounts in ^{42}Sc and ^{42}Ca. Thus these wavefunctions are now written

$$\begin{aligned}\psi_i(^{42}Sc) &= b_0\psi_0 + b_1\psi_1 \\ \psi_{f0}(^{42}Ca) &= a_0\psi_0 + a_1\psi_1 \\ \psi_{f1}(^{42}Ca) &= a_1\psi_0 - a_0\psi_1,\end{aligned} \tag{15}$$

with a_0 and b_0 both being close to unity. The Fermi matrix element between ground states becomes

$$\begin{aligned}\langle\psi_{f0}(^{42}Ca)|\tau_+|\psi_i(^{42}Sc)\rangle &= a_0b_0\langle\psi_0|\tau_+|\psi_0\rangle + a_1b_1\langle\psi_1|\tau_+|\psi_1\rangle \\ &= \sqrt{2}(a_0b_0+a_1b_1), \\ |\langle\psi_{f0}(^{42}Ca)|\tau_+|\psi_i(^{42}Sc)\rangle|^2 &\simeq 2\left(1-(a_1-b_1)^2\right),\end{aligned} \tag{16}$$

using $a_0^2+a_1^2=1$, $b_0^2+b_1^2=1$. Further the matrix element between ^{42}Sc ground state and the ^{42}Ca excited state is

$$\begin{aligned}\langle\psi_{f1}(^{42}Ca)|\tau_+|\psi_i(^{42}Sc)\rangle &= a_1b_0\langle\psi_0|\tau_+|\psi_0\rangle - a_0b_1\langle\psi_1|\tau_+|\psi_1\rangle \\ &= \sqrt{2}(a_1b_0-a_0b_1), \\ |\langle\psi_{f1}(^{42}Ca)|\tau_+|\psi_i(^{42}Sc)\rangle|^2 &\simeq 2(a_1-b_1)^2,\end{aligned} \tag{17}$$

and is no longer zero. If we write the ground state to ground state matrix element squared as: $|M_{\rm V}^0|^2 = 2(1-\delta_{C1}^0)$, and the ground state to excited state matrix element squared as: $|M_{\rm V}^1|^2 = 2\delta_{C1}^1$, then for two-state mixing the corrections are equal:

$$\delta_{C1}^0 = \delta_{C1}^1 = (a_1-b_1)^2. \tag{18}$$

This is a specific result for two-state mixing, in general they would not be equal. Further the correction δ_{C1} depends on the *difference* in the degree of isospin mixing in ^{42}Sc relative to ^{42}Ca. This is clearly evident if we use perturbation theory to estimate the small amplitudes a_1 and b_1. Let $V_C(^{42}Ca)$ and $V_C(^{42}Sc)$ be the Coulomb and other charge-dependent forces operative in ^{42}Ca and ^{42}Sc respectively, then

$$\delta_{C1}^0 = \delta_{C1}^1 = \frac{\left[\langle\psi_1|V_C(^{42}Ca)|\psi_0\rangle - \langle\psi_1|V_C(^{42}Sc)|\psi_0\rangle\right]^2}{(E_1-E_0)^2}, \tag{19}$$

TABLE 2. Experimental branching ratios, R, for non-analogue Fermi transitions, and values of δ_{C1} from experiment and theory.

Nuclide	Expt[a] R(ppm)	Expt[a] $\delta^1_{C1}(\%)$	Theory $\delta^1_{C1}(\%)$	Theory $\delta^0_{C1}(\%)$
^{38m}K	< 19	< 0.28	0.096	0.100
^{42}Sc	59(14)[b]	0.040(9)	0.041	0.049
^{46}V	39(4)	0.053(5)	0.046	0.087
^{50}Mn	< 3	< 0.016	0.051	0.068
^{54}Co	45(6)	0.035(5)	0.037	0.045

[a] From Hagberg *et al.* [15]
[b] Daehnick *et al.* [19] averaged with earlier results [20–22]

where $E_1 - E_0$ is the energy separation between the excited- and ground- 0^+ states from the charge-independent Hamiltonian. Thus δ_{C1} is inversely proportional to the square of this energy difference.

In a shell-model calculation it is notoriously difficult to obtain correctly the experimental $E_1 - E_0$ energy separation. This is because the $|4p\text{-}2h\rangle$ is a deformed state, while the $|2p\rangle$ is a spherical state and these two distinct aspects are difficult to realise in a truncated calculation in a spherical basis. Thus in the calculations we are about to describe, the values of δ^0_{C1} and δ^1_{C1} obtained are both corrected using

$$\delta_{C1} = \delta^{\text{calc}}_{C1} \times \frac{(E_1 - E_0)^2_{\text{calc}}}{(E_1 - E_0)^2_{\text{expt}}}. \tag{20}$$

In this way, we believe we are adjusting the δ_{C1} value approximately for the imperfections of the underlying strong-interaction Hamiltonian and the necessity of using model-space truncations.

Here we discuss calculations for ^{38m}K, ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co, which were recomputed recently to compare with the excited state non-analogue Fermi transitions measured by Hagberg *et al.* [15]. In each case we use the largest model space practicable in a proton-neutron (pn) basis. For ^{38m}K and ^{42}Sc this involved orbitals in both the (s, d) and (p, f) shells and the effective interaction constructed by Warburton, Becker, Millener and Brown (WBMB) [16] was used for the underlying strong interaction. For ^{46}V, ^{50}Mn and ^{54}Co, the orbitals span the (p, f) shell and the strong interaction was taken to be FPMI3 from Richter *et al.* [17]. The single-particle energies were fixed from experimental values at the closed-shell-plus-one configuration.

The Coulomb and charge-dependent interaction terms to be added were constrained so that the ground-state masses of the isotriplet of states, $T_z = -1, 0, +1$, were fitted. The isobaric multiplet mass equation (IMME) writes these masses as

$$M(T_z) = a + bT_z + cT_z^2, \tag{21}$$

where the coefficient a represents the result from a charge-independent Hamiltonian, and b and c are charge-dependent corrections. Specifically

$$\begin{aligned} b &= \left(M(T_z = +1) - M(T_z = -1)\right)/2 \\ c &= \left(M(T_z = +1) + M(T_z = -1) - 2M(T_z = 0)\right)/2, \end{aligned} \tag{22}$$

so b is governed by the difference between pp and nn forces, and c by the difference between the pn and nn forces. Our strategy was to multiply the two-body Coulomb matrix elements by a constant factor so that the b-coefficient of the IMME is reproduced, and likewise the pn matrix elements are multiplied by a constant factor to reproduce the c-coefficient. This strategy of adjusting the strength of the Coulomb and charge-dependent nuclear forces to reproduce the IMME equation was pioneered by Ormand and Brown [18].

The results of these calculations for δ^0_{C1} and δ^1_{C1} for nuclei ^{38m}K, ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co are given in Table 2. Only the δ^0_{C1} value is needed for the analysis of the superallowed Fermi data. However, the companion δ^1_{C1} value can be subjected to an experimental test. If the Fermi transition to an excited 0^+, non-analogue, state can be determined then the measured branching ratio, R, is proportional to δ^1_{C1}. These branching ratios are very small, parts per million (ppm), so their measurement requires a dedicated effort, and the work of

TABLE 3. Calculated Coulomb correction, δ_C in percent units.

Nuclide	Towner-Hardy[a] δ^0_{C1}(%)	δ_{C2}(%)	Ormand-Brown[b] δ^0_{C1}(%)	δ_{C2}(%)	Adopted Value[c] $\delta_C = \delta_{C1} + \delta_{C2}$(%)
^{10}C	0.006	0.17	0.04	0.11	0.16(3)
^{14}O	0.004	0.28	0.01	0.14	0.22(3)
^{26m}Al	0.057	0.27	0.01	0.29	0.31(3)
^{34}Cl	0.024	0.62	0.06	0.51	0.61(3)
^{38m}K	0.100	0.54	0.11	0.48	0.62(3)
^{42}Sc	0.049	0.35	0.11	0.31	0.41(3)
^{46}V	0.087	0.36	0.09	0.29	0.41(3)
^{50}Mn	0.068	0.40	0.02	0.33	0.41(3)
^{54}Co	0.045	0.56	0.04	0.40	0.52(3)

[a] Refs. [9,10]
[b] Ref. [11]
[c] Average of Towner-Hardy and Ormand-Brown values; assigned uncertainties reflect the *relative* scatter between these calculations.

the Chalk River group in this regard is described in Hagberg *et al.* [15]. If t_0 is the partial half-life for the ground-state decay and t_1 the the partial half-life to the excited 0^+ state, then

$$R = \frac{t_0}{t_1} = \frac{f_1}{f_0}\frac{f_0 t_0}{f_1 t_1} = \frac{f_1}{f_0}\frac{2\delta^1_{C1}}{2(1-\delta^0_{C1})} \simeq \frac{f_1}{f_0}\delta^1_{C1}, \tag{23}$$

where f_0 and f_1 are phase space integrals for the ground state and excited state respectively. Table 2 lists the experimental value of R and δ^1_{C1}. The comparison between theory and experiment is exceptional in all cases except ^{50}Mn and so provides a lot of confidence that the companion δ^0_{C1} values used in the superallowed Fermi data analysis are reasonable. The ^{50}Mn case is interesting in that the experiment was unable to locate a branch to an excited 0^+ state in the expected energy region and so deduced that any such branching ratio would be less than 3 ppm.

The δ^0_{C1} values for lighter superallowed Fermi emitters, ^{10}C, ^{14}O, ^{26m}Al and ^{34}Cl were obtained in a similar manner, as described in Towner [10]. The $\delta_{C1} = \delta^0_{C1}$ values are quite small, of the order of 0.1%, but seem under reasonable control. The larger δ_{C2} values associated with radial overlap integrals, potentially can have more uncertainty. We have found that our results based on using Saxon-Woods radial functions lead to systematically larger δ_{C2} values than the calculations of Ormand and Brown [11] with Hartree-Fock functions. We therefore allow for this systematic difference in our data analysis by using average values and introducing a systematic uncertainty to $\overline{\mathcal{F}t}$ (see equation (4)). In Table 3 we give our δ_{C1} and δ_{C2} values, together with those of Ormand and Brown, and our adopted final numbers, $\delta_C = \delta_{C1} + \delta_{C2}$. These are the values that appear in Table 1 and are used to analyze the world data for superallowed $0^+ \to 0^+$ decays.

FUTURE DIRECTIONS

With the experimental evidence so far completely in support of the calculated values for δ_C, the current world data on superallowed $0^+ \to 0^+$ beta decay are tantalizingly close to a result in definitive disagreement with CKM unitarity. Naively one might expect such a situation would prompt an urgent new round of experiments with the goal of further reducing the quoted uncertainty in $\mid V_{ud} \mid$ but, unfortunately, the next step cannot be so straightforward. As we have already noted, the error bar associated with $\mid V_{ud} \mid$ in equation (7) is now dominated by uncertainties in the calculated correction terms. Any improvements in precision made within the existing body of experimental data will be effectively lost once the results are applied to the unitarity test so long as there are no improvements in the calculations.

Clearly of highest priority in future must be to increase the precision of the correction terms, particularly $\Delta^{\rm V}_{\rm R}$, which is the largest contributor to the uncertainty of $\mid V_{ud} \mid$. Its importance is manifest not only in the unitarity test based on superallowed $0^+ \to 0^+$ beta decay but also in any tests based on neutron or pion decays. To date, the experimental data on these non-nuclear decays are considerably less precise [4] than those on the $0^+ \to 0^+$ transitions but, year by year, improvements are being made in the neutron-decay measurements largely motivated by the prospect of a unitarity test unfettered with the structure-dependent

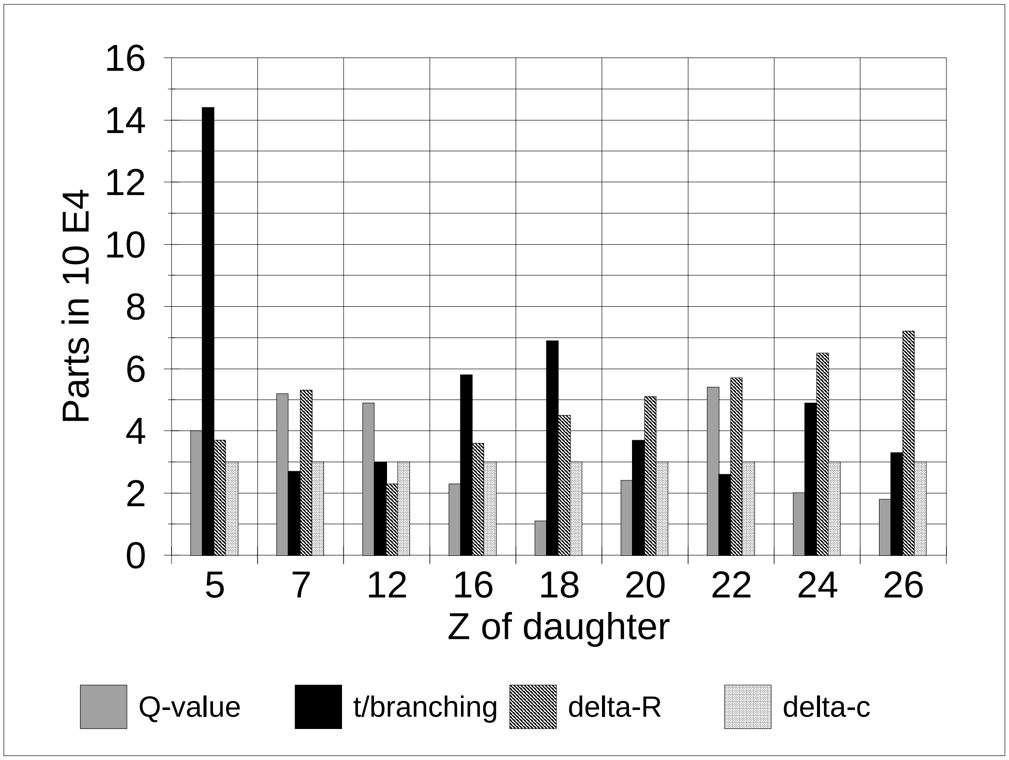

FIGURE 2. Contributions from experiment and theory to the overall $\mathcal{F}t$-value uncertainty for each superallowed transition listed in Table 1.

Coulomb correction, δ_C, which vanishes for the neutron. In spite of this simplification in the neutron decay, however, any potential advantage to the unitarity test is lost unless $\Delta_{\rm R}^{\rm V}$ can be calculated with greater precision, since *all* determinations of $|V_{ud}|$ depend directly on $\Delta_{\rm R}^{\rm V}$.

Until such time as the $\Delta_{\rm R}^{\rm V}$ calculation is refined and the neutron decay measurements rival the precision of the nuclear results, the best hope for improvements to the unitarity test lies in increasing our confidence in the calculated values of δ_C. Though a reasonable estimate of the uncertainty in δ_C has been incorporated into the derivation of $|V_{ud}|$ (see Tables 1 and 3), there is no doubt that the dominant role of nuclear structure in the calculation of δ_C leaves some people questioning whether the real uncertainty is larger than the one actually quoted. Under the circumstances, any experiment that can probe the veracity of the δ_C calculations will make a valuable contribution to the whole problem. There are at least three different approaches that can be taken in devising such experiments.

First, improvements can be sought in results for the nine superallowed transitions whose ft-values are already known to within a fraction of a percent. This would not be a fruitless endeavour. It is certainly true that, given the large quantity of careful measurements now contributing to the content of Table 1, there is little chance that the central value of $\overline{\mathcal{F}t}$ will be changed significantly by a few more. And, it is also true that improvements in the experimental uncertainties will not be directly reflected in a reduced uncertainty for $|V_{ud}|$ unless the $\Delta_{\rm R}^{\rm V}$ correction has been improved too. But, the test of CVC can be made more demanding as the experimental precision is increased and, to the extent that the $\mathcal{F}t$-values continue to agree with one another, this would demonstrate at the same time the reliability of the δ_C calculations, which compensate for the transition-to-transition variations evident in the uncorrected ft-values. Of course, it is only the *relative* values of δ_C that can be tested by this method, but it would be a pathological fault indeed that could calculate in detail the required variations in δ_C while failing to obtain their *absolute* values to comparable precision.

The various experimental and theoretical contributions to the $\mathcal{F}t$-value uncertainties are shown in Fig. 2 for the nine superallowed transitions whose ft values are known to within a fraction of a percent. If we accept that it is valuable for experiment to be at least a factor of two more precise than the calculations for δ_R and δ_C (*relative*

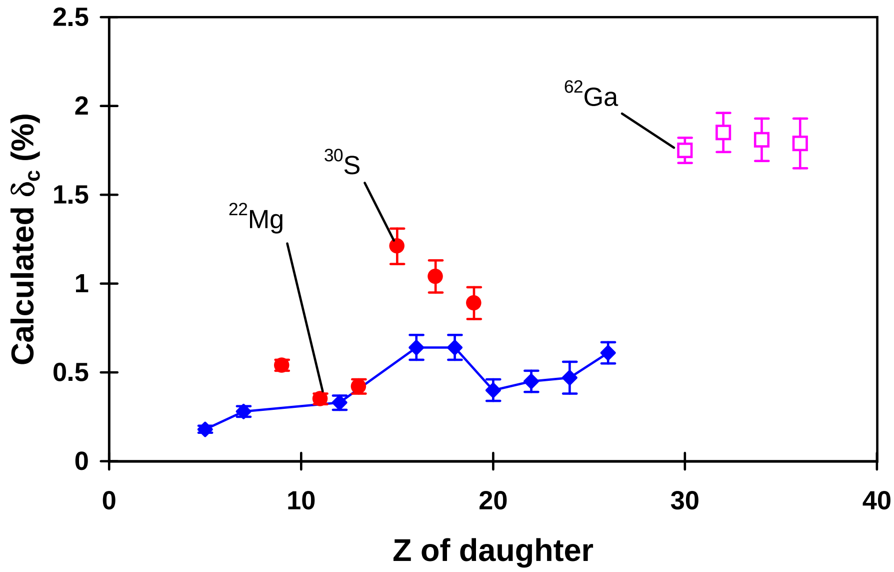

FIGURE 3. Calculated δ_C values plotted as a function of the Z of the daughter nucleus. The solid diamonds joined by the line represent the values for the nine well known superallowed emitters listed in Table 1. The circles are for the $T_z = -1$ emitters between ^{18}Ne and ^{38}Ca; while the open squares are for the $T_z = 0$ cases from ^{62}Ga to ^{74}Rb.

uncertainties only), then an examination of Fig. 2 shows that the Q-values for ^{10}C, ^{14}O, ^{26m}Al and ^{46}V, the half-lives of ^{10}C, ^{34}Cl and ^{38m}K, and the branching ratio for ^{10}C can all bear improvement. Such improvements will soon be feasible. The Q-values will reach the required level (and more) as mass measurements with new on-line Penning traps become possible; half-lives will likely yield to measurements with higher statistics as high-intensity beams of separated isotopes are developed for radioactive-beam facilities; and, finally, an improved branching-ratio measurement on ^{10}C has already been made with Gammasphere and simply awaits analysis [23].

Another experimental approach to testing δ_C is offered by the possibility of increasing the number of superallowed emitters accessible to precision studies. The greatest attention recently has been paid to the $T_z = 0$ (odd-odd) emitters with $A \geq 62$, since these nuclei are expected to be produced at new radioactive-beam facilities, and their calculated Coulomb corrections, δ_C, are predicted to be large [11,13,24], as is illustrated in Fig. 3. In principle, then, they could provide a valuable test of the accuracy of δ_C calculations. It is likely, though, that these heavy emitters will not provide ft-values with sufficient precision to be useful directly in extracting competitive $\mathcal{F}t$-values in the near future. All of the well known emitters listed in Table 1, with the exception of ^{10}C, have the special advantage that the superallowed branch from each is by far the dominant transition in its decay ($> 99\%$). This means that the branching ratio for the superallowed transitions can be determined to high precision from relatively imprecise measurements of the other weak transitions, which can simply be subtracted from 100%. In contrast, the decays of the heavier $T_z = 0$ emitters – nuclides such as ^{62}Ga, ^{66}As, ^{70}Br and ^{74}Rb – will be of considerably higher energy and each will therefore involve several allowed transitions of significant intensity in addition to the superallowed transition. Branching-ratio measurements will thus be very demanding, particularly with the limited radioactive-beam intensities likely to be available initially for these rather exotic nuclei. Lifetime measurements will be similarly constrained by statistics. As to the Q_{EC} values, even Penning traps will be hard pressed to produce the required precision of a few parts in 10^9 for the masses of these short-lived ($t_{1/2} \leq 100$ ms) nuclides.

More immediately achievable among these heavier superallowed emitters are measurements of the type described in the preceding section, in which non-analogue Fermi transitions were observed [15] and compared with model calculations as a test of the techniques used in calculating δ_C. Such measurements would yield important information on Coulomb mixing and they would be an important prerequisite for any serious attempt to obtain $\mathcal{F}t$-values in this region of nuclei. Another important prerequisite would be the experimental determination of the coefficients of the IMME for all relevant 0^+ $T = 1$ states. These demanding experiments are interesting in their own right but, as preliminaries to future $\mathcal{F}t$-value determinations, they are especially important because

model calculations in this region of rapid shape changes are likely to be far less reliable than they are in the (s, d) and (p, f) shells. Without the constraints provided by these experiments – and possibly even with them – tests of δ_C from the measured $\mathcal{F}t$-values could end up reflecting more about the limitations of the nuclear models used than about the underlying physics of the weak interaction. As a first probe of these issues, an investigation of non-analogue Fermi transitions from ^{62}Ga is already underway [25]. More will undoubtedly follow.

In the near future, the most promising experimental approach that can actually increase the number of precisely measured $\mathcal{F}t$-values is to study the $T_z = -1$ superallowed emitters with $18 \leq A \leq 38$. There is good reason to explore them. For example, as shown in Fig. 3, the calculated value [9] of δ_C for ^{30}S decay, though smaller than the δ_C's expected for the heavier nuclei, is actually 1.2% – about a factor of two larger than for any other case currently known – while ^{22}Mg has a very low value of 0.35%. If the ft-values for these two nuclei can be determined to a precision of a few tenths of a percent or better, and the large predicted difference is confirmed, then it will do much to increase our confidence in the calculated Coulomb corrections. This would be especially convincing since the calculation involves the same model space as was used for the presently known cases. To be sure, these decays will provide an experimental challenge, particularly in the measurement of their branching ratios, but the required precision should be achievable with isotope-separated beams that are currently available. In fact, such experiments are also in their early stages at the Texas A&M cyclotron [26].

CONCLUSIONS

The current world data on superallowed $0^+ \rightarrow 0^+$ beta decays lead to a self-consistent set of $\mathcal{F}t$-values that agree with CVC but differ provocatively, though not yet definitively, from the expectation of CKM unitarity. There are no evident defects in the calculated radiative and Coulomb corrections that could remove the problem, but suspicion continues to fall on the calculations of Coulomb mixing, which depend sensitively on the details of nuclear structure. If any progress is to be made in firmly establishing (or eliminating) the discrepancy with unitarity, additional experiments are required that focus on this issue. We have indicated what some relevant nuclear experiments might be, and have particularly emphasized that experiments to measure the ft-values of heavy $T_z = 0$ odd-odd superallowed emitters with $A \geq 62$, which have been proposed for new radioactive-beam facilities, are very difficult and should be preceded by measurements in the same mass region of non-analogue Fermi decays and IMME coefficients.

On the theoretical side, the most important requirement for all tests of CKM universality that depend upon V_{ud} is an improved determination of the nucleus-independent radiative correction, $\Delta_{\rm R}^{\rm V}$. It is not only the results from nuclear superallowed decays that must be subjected to this correction term, but also the results from the neutron and pion decays if they also are to be used to extract V_{ud}. Though the latter decays are currently known with less precision than the nuclear decays, one can reasonably expect them to improve significantly over the next decade. Thus, it is of highest priority to reduce the uncertainty currently attached to the calculation of $\Delta_{\rm R}^{\rm V}$. That having been stated, it must be noted that nuclear decays will also require more reliable δ_C calculations to remain competitive, especially if the uncertainties are reduced on $\Delta_{\rm R}^{\rm V}$. In any case, improved model calculations of nuclear structure and Coulomb mixing in nuclei with $N \simeq Z$ and $A \geq 62$ are an important requirement for the future if ft-value measurements are to be attempted in this region.

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REFERENCES

1. Hardy, J.C., Towner, I.S., Koslowsky, V.T., Hagberg, E., and Schmeing, H., *Nucl. Phys.* A**509**, 429 (1990).
2. Towner, I.S., and Hardy, J.C., in *Symmetries and Fundamental Interactions in Nuclei*, eds. Henley, E.M., and Haxton, W.C., Singapore: World-Scientific, 1995, pp.183-249.
3. Caso, C., *et al.*, *Eur. Phys. J.* C**3**, 1 (1998).
4. Towner, I.S., and Hardy, J.C., in *Proceedings of the V Int. Symp. on Weak and Electromagnetic Interactions* Singapore: World-Scientific, 1998, to be published.
5. Sirlin, A., *Phys. Rev.* **164**, 1767 (1967).
6. Sirlin, A., *Phys. Rev.* D**35**, 3423 (1987); Sirlin, A., and Zucchini, R., *Phys. Rev. Lett.* **57**, 1994 (1986).
7. Jaus, W., and Rasche, G., *Phys. Rev.* D**35**, 3420 (1987).

8. Towner, I.S., *Phys. Lett.* B**333**, 13 (1994).
9. Towner, I.S., Hardy, J.C., and Harvey, M., *Nucl. Phys.* A**284**, 269 (1977).
10. Towner, I.S., in *Symmetry Violations in Subatomic Physics*, eds. Castel, B., and O'Donnel, P.J., Singapore: World-Scientific, 1989, p.211.
11. Ormand, W.E., and Brown, B.A., *Phys. Rev.* C**52**, 2455 (1995).
12. Sirlin, A., in *Precision Tests of the Standard Electroweak Model*, ed. Langacker, P., Singapore: World-Scientific, Singapore, 1994.
13. Sagawa, H., van Giai, N., and Suzuki, T., *Phys. Rev.* C**53**, 2163 (1996).
14. Navrátil, P., Barrett, B.R., and Ormand, W.E., *Phys. Rev.* C**56**, 2542 (1997).
15. Hagberg, E., Koslowsky, V.T., Hardy, J.C., Towner, I.S., Hykawy, J.G., Savard, G., and Shinozuka, T., *Phys. Rev. Lett.* **73**, 396 (1994).
16. Warburton, E.K., Becker, J.A., Brown, B.A., and Millener, D.J., *Ann. Phys. (N.Y.)* **187**, 471 (1988).
17. Richter, W.A., van dar Merwe, M.G., Julies, R.E., and Brown, B.A., *Nucl. Phys.* A**523**, 325 (1991).
18. Ormand, W.E., and Brown, B.A., *Nucl. Phys.* A**440**, 274 (1985).
19. Daehnick, W.W., and Rosa, R.D., *Phys. Rev.* C**5**, 1499 (1985).
20. Sandorfi, A.M., Lister, C.J., Alburger, D.E., and Warburton, E.K., *Phys. Rev.* C**22**, 2213 (1980).
21. DelVecchio, R.M., and Daehnick, W.W., *Phys. Rev.* C**17**, 1809 (1978).
22. Ingalls, R.D., Overley, J.C., and Wilson, H.S., *Nucl. Phys.* A**293**, 117 (1977).
23. Freedman, S.J., private communication.
24. Hardy, J.C., in *Proceedings of the Workshop on the Production and Use of Intense Radioactive Beams at the Isospin Laboratory, 1992*, ed. J.D. Garrett (Joint Institute for Heavy Ion Research, Oak Ridge, Tennessee: Conf. 9210121) pg. 51.
25. Hyman, B.C., Azhari, A., Gagliardi, C.A., Hardy, J.C., Sandu, T., Tang, X.D., Trache, L. and Tribble, R.E., in *Progress in Research, 1997/1998, Cyclotron Institute, Texas A& M University*, pg. I-31.
26. Hardy, J.C., Azhari, R. Burch, A., Gagliardi, C.A., Lipnik, P., Mayes, E., Trache, L. and Tribble, R.E., in *Progress in Research, 1997/1998, Cyclotron Institute, Texas A& M University*, pg. I-34.